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DYNAMIC MODELING AND CONTROLLER DESIGN FOR NUCLEAR POWER PLANT

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ABSTRACT

Higher order mathematical modeling and discrete fast output sampling control synthesis for PHWR (Pressurized Heavy Water Reactor)-type nuclear power plant is presented in this paper. The nonlinear dynamic model of PHWR power reactor is developed based on reactor, logarithmic amplifier, moderator level control valve and reactivity system. The nonlinear model is characterized by 15 state variables and one input. This higher order nonlinear model is linearized at full power operating point. A standard higher order state space model is realized for controller design purpose. An advanced discrete controller is synthesized for state space model of PHWR using fast output sampling methodology. Noise sensitivity is a typical problem with this type of controller. In this research work, a small deviation is allowed in order to reduce the noise sensitivity. The problems of noise sensitivity and poor output error dynamics are addressed and a multi-objective optimization problem is solved using LMI (Linear Matrix Inequalities) formulation. Fast output feedback gains are computed using LMI tool for MATLAB. An advanced fast output sampling controller is evaluated for power maneuvering from 270 MW_{th} to 310 MW_{th} on the nuclear power plant in Pakistan and found highly efficient and satisfactory within the control design constraints.

Key words: *Moderator Level Dynamics, Reactor Power Control, PHWR, Nuclear Power Plant, Fast Output Sampling technique, LMI*

INTRODUCTION

Power control system in pressurized heavy water reactor type nuclear reactor is controlled by moderator reactivity changes, control rod reactivity changes and soluble poison reactivity changes. In existing power control system, compensator based hard wired control logic is used. This compensator based control system uses number of checks and variable saturation limits on controller signals based on plant power demand conditions. Modifications in control system and design changes with time are not possible with this hard wired controller. This hard wired controller is interfaced with real time nuclear power plant. Thus, in order to design a new controller for the utility, model development for nuclear power reactor is an essential requirement. This model based controller design will not only useful for control modifications in the existing utility but also serve a sound basis for advanced control design for future PHWR type nuclear plants in Pakistan. An advanced control design for nuclear power reactor should be so designed that it incorporates both linear power control and logarithmic rate power control. Dynamic modeling, simulation and advanced controller design for nuclear power plants are not much investigated by the researchers. State feedback assisted classical control approach for Research reactor and pressurized water reactor (PWR)-type nuclear power is addressed in[1]. The robust multivariable feed forward / feedback controller is designed for boiling water reactor (BWR)-type nuclear power plant in[2]. The Monte Carlo method is used for the estimation of research reactor dynamics in[3]. In[4], a nonlinear model of Russian pressurized water reactor type nuclear power plant has been investigated. A first principle based PHWR model was proposed for spatial control and different controllers like LQR controller, periodic output feedback controller and multi-rate sliding mode controller have been addressed in[5-8].

In[5-8], Liquid Zone Control (LZC) scheme has been addressed for reactor power control. The dynamics and control of PHWR under consideration is

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entirely different from PHWR considered in[5-8]. In this paper, new modeling and control issues of logarithmic reactor power and logarithmic rate reactor power are specially addressed that are very essential for reactor criticality analysis and reactivity transients. The modeling issue of geometrical variations in reactor calandria shape is also incorporated for the first time in this paper which has never been touched by the researchers. In this paper, an advanced control system design is proposed for an operating unit in Pakistan[9] with novel emphasis on moderator level dynamics, nonlinear logarithmic amplifier, moderator level control (MLC) system and multi-objective LMI problem formulation for less noise sensitivity, fast output error dynamics and efficient control.

MATHEMATICAL MODEL DEVELOPMENT: PHWR

In this paper, an operating unit of 432 MW_{th} power is considered situated in Pakistan. The control system of the reactor is schematically shown in Figure 1. It consists of reactor (the main control element), nuclear instruments (sensors, comparing and other circuits), error amplifier and the moderator system comprising of moderator level control valve and moderator flow control. It should be recognized that the development of mathematical model representation of such a complicated system would not be easy because of highly non-linear elements such as reactor, logarithmic amplifier, control valve and reactivity system. Two outputs are measured namely linear power amplifier and logarithmic rate power amplifier outputs. A control signal is constructed based on these two outputs and control signal actuates the valve which controls the moderator level in the reactor and hence the power output from the reactor. The closed loop framework of advanced control system for PHWR is shown in Figure 1. In closed loop framework, two loops are identified; one loop is designed for linear reactor power control while second loop is designed for transient control in reactor criticality.

In this research work, it is assumed that the concentration of soluble poison and position of control rods are fixed and reactor power is accomplished with moderator control valve. It means all the control rods are out of the reactor core and are not moving. This is required to ensure the reactor safety and operation of control rods in a predefined band so that control rods can be moved down into the core when moderator level between 180 to 186 inches. The concentration of soluble poison is fixed for hot reactor

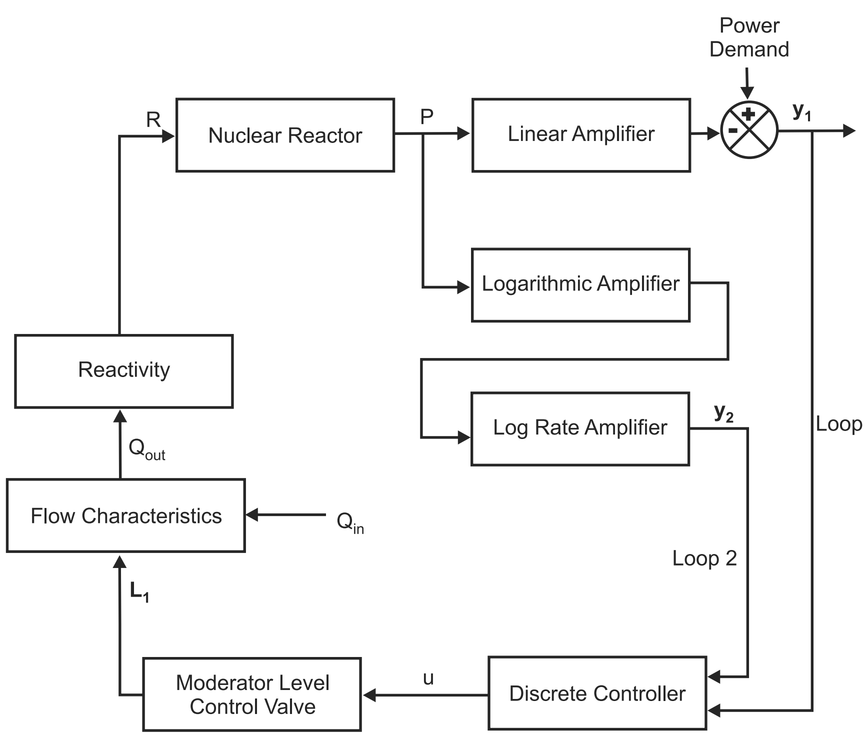

Figure 1: Framework of advanced control system for PHWR NPP

operating at certain power level. The concentration of soluble poison affects the reactor criticality for a cold reactor start-up from zero power and this would little bit sluggish the reactor power rise rate.

A detailed non-linear dynamic modeling, linearization and state space realization of PHWR model is presented in the following sections.

Non-linear Dynamic Model of PHWR

All the PHWR model variables and parameters are defined in Tables 1 and 2 respectively. The values of all physical parameters of pressurized water reactor are mentioned in Table 3.

Following differential equations describe the PHWR under consideration and its control mechanism:

Table 1: PHWR Model Variables with Definitions

Variables	Definitions
P	Reactor Power (MW_{th})
R	Reactivity (mk)
C_i	Concentrations of the delayed neutron precursors (i=1,2,…,6)
H	Moderator Height (inches)
Q_{IN}	Inflow of Moderator (liters/min)
Q_{OUT}	Outflow of Moderator (liters/min)
V_L	Linear Power Amplifier Output (Volts)
V_{LG1}	Logarithmic Power Amplifier Output (Volts)
V_{LG2}	Derivative of Logarithmic Power Amplifier Output (Volts/sec)
V_{LGR1}	Logarithmic Rate Power Amplifier Output (Volts)
V_{LGR2}	Derivative of Logarithmic Rate Power Amplifier Output (Volts/sec)
L_1	Moderator Control Valve Position (%)
L_2	Derivative of Moderator Control Valve Position (%/sec)
Q_M	Moderator Flow Control Input (Volts)

Table 2. PHWR Model Parameters with Definitions

Parameters	Definitions
β_i	Delayed Neutron Fractions (i=1,2,…,6)
λ_i	Decay Constants for Precursors (i=1,2,…,6)
l	Prompt Neutron Life Time (sec)
K_C	Calandria Shape Constant
A_C	Area of Cross-section of Calandria (cm^2)
K_L	Linear Power Amplifier Gain
τ_L	Linear Power Amplifier Time Constant (sec)
K_{LG}	Logarithmic Power Amplifier Gain
τ_{LG1}	Logarithmic Power Amplifier Time Constant at Low Power (sec)
τ_{LG2}	Logarithmic Power Amplifier Time Constant at High Power (sec)
K_{LGR}	Logarithmic Rate Power Amplifier Gain
τ_{LGR1}	Logarithmic Rate Power Amplifier Time Constant at Low Power (sec)
τ_{LGR2}	Logarithmic Rate Power Amplifier Time Constant at High Power (sec)
ς	Damping Coeficient of Control Valve
w_n	Natural Frequency of Control Valve (rad/sec)
K_V	Sensitivity of Control Valve (%/ Volts)
K_F	Flow Coefficient for Moderator Control Valve (cm^3/%)

Pressurized Heavy Water Reactor

The reactor power output and precursor concentrations are represented by the following first order coupled differential equations based on point kinetic model [3]:

$$\frac{dP}{dt} = \frac{\rho - \beta}{l} P + \sum \lambda_i C_i \quad and \qquad (1)$$

Table 3: Physical Parameters of 432 MW_{th} PHWR Model

Parameters	Values
l	8.37×10^{-4} sec
λ_1	3.623 sec^{-1}
λ_2	1.348 sec^{-1}
λ_3	0.303 sec^{-1}
λ_4	0.127 sec^{-1}
λ_5	0.032 sec^{-1}
λ_6	0.012 sec^{-1}
β_1	0.23×10^{-3}
β_2	0.599×10^{-3}
β_3	0.192×10^{-2}
β_4	0.972×10^{-3}
β_5	0.111×10^{-2}
β_6	0.149×10^{-3}
K_C	68.2×10^{-3}
A_C	136 cm^2
ς	0.7
w_n	10 rad/sec
K_V	12.05%/Volt
K_F	11 cm^3/%

$$\frac{dC_i}{dt} = \frac{\beta_i}{l} P - \sum \lambda_i C_i \quad i = 1, 2, \ldots, 6 \tag{2}$$

where $\beta = \sum_{i=l}^{6} \beta_i$

In PHWR type nuclear power plant, U-235 is used as nuclear fuel which has twenty four fission fragments. Each fission fragment is called precursor which is responsible for the production of delayed neutrons. Amongst twenty four delayed neutrons, six dominant delayed neutrons are considered of close half lives. In this paper, six delayed neutron groups are considered. Thus, a 7^{th} order reactor model is obtained. This higher order model is a time dependent reactor model without any effect of the shape of reactor core.

Reactivity

At any instant the reactivity in the reactor is a function of heavy water level dynamics. Change in the heavy water level is affected by the outflow variation in the calandria keeping the inflow constant. The reactivity dynamics and moderator level dynamics in the calandria are mathematically formulated based on the design data of the pressurized heavy water reactor[9] and represented by the following equations:

$$\rho = \rho_0 + K_C \left(\frac{\pi^2}{H_0^2} - \frac{\pi^2}{H^2} \right) \tag{3}$$

where $H_0 = 35$ inches and $\rho_0 = 0$, at steady state.

$$\frac{dH}{dt} = \frac{Q_{IN} - Q_{OUT}}{A_C} \tag{4}$$

δQ is computed using the following relationship:

$$\delta Q = K_F \delta L_1 \tag{5}$$

Level Control Valve

The dynamics of opening of equal percentage type control valve can be approximated by following second order transfer function as:

$$\frac{L}{Q_M} = \frac{K_V}{s^2 + 2\varsigma\omega + \omega_n^2} \tag{6}$$

where the sensitivity of control valve is computed from closed loop plant data while damping coefficient and natural frequency of control valve are computed using characteristic curve of control valve as reported in[9]. The frequency domain transfer function described in Equation (6) is transformed into time domain using inverse Laplace transformation. Therefore, the dynamics of the opening of equal percentage type control valve is governed by the following second order differential equation as:

$$\frac{d^2 L_1}{dt^2} + 2\varsigma\omega_n \frac{dL_1}{dt} + \omega_n^2 L_1 = L_V Q_M \tag{7}$$

If $\dot{L}_1 = L_2$ and $\ddot{L}_1 = \dot{L}_2$ then Equation (7) can be written as:

$$\frac{dL_1}{dt} = L_2 \tag{8}$$

$$\frac{dL_2}{dt} = K_V u - 2\varsigma\omega_n L_2 - \omega_n^2 L_1 \tag{9}$$

Linear Amplifier

Values of all gain constants and time constants of PHWR linear, logarithmic and logarithmic rate power amplifiers are given in Table 4.

Power of reactor is converted into voltage signal using linear amplifier. The voltage signal is a damped differential of reactor power which is simply a lag unit. Linear amplifier and power relation is expressed in terms of following differential equation:

$$\frac{dV_L}{dt} = \frac{K_L}{\tau_L} P - \frac{V_L}{\tau_L} \tag{10}$$

Logarithmic Amplifier

Frequency domain transfer functions of logarithmic amplifier and logarithmic rate amplifier are addressed in[10]. Based on the frequency domain transfer functions, logarithmic amplifier and logarithmic rate amplifier are modeled and presented in differential equations form.

Logarithmic amplifier is a very complex electronic unit which is used for the logarithmic power measurement. It produces an output in decades which is converted into voltage signal. It is primarily a requirement of logarithmic rate power.

Table 4: Gain Constants and Time Constants for 432 MW_{th} PHWR Amplifiers

Constants	Values
K_L	0.1
τ_L	0.018
K_{LG}	200
τ_{LG1}	2.2
τ_{LG2}	6
K_{LGR}	20
τ_{LGR1}	0.3
τ_{LGR2}	10

In logarithmic amplifier, the concept of logarithmic of linear reactor power is used. The relationship between the logarithmic of linear reactor power (V_{LL}) expressed in MW_{th} and voltage is given as:

$$V_{LL} - \log_{10}(10^4 P) \tag{11}$$

In frequency domain, the logarithmic amplifier output voltage can be expressed by a linear gain cascaded with a standard second order transfer function given as[10]:

$$\frac{V_{LG1}}{V_{LL}} = \frac{K_{LG}\omega_{nLG}^2}{S^2 + 2\varsigma_{LG}\omega_{nLG} + \omega_{nLG}^2} \tag{12}$$

where damping coefficient and natural frequency of logarithmic amplifier are defined as:

$$\omega_{nLG} = \frac{1}{\sqrt{\tau_{LG1}\tau_{LG2}}}$$

and

$$\varsigma_{LG} = \frac{1}{2}\frac{(\tau_{LG1} + \tau_{LG2})}{\sqrt{\tau_{LG1}\tau_{LG2}}}$$

Substituting the value of (V_{LL}) from Equation (11) in Equation (12) and taking inverse Laplace transform, the logarithmic amplifier output voltage and power relation can be expressed in terms of following second order differential equations as:

$$\frac{d^2V_{LG1}}{dt^2} + \frac{(\tau_{LG1} + \tau_{LG2})}{(\tau_{LG1}\tau_{LG2})}\frac{dV_{LG1}}{dt} + \frac{1}{(\tau_{LG1}\tau_{LG2})} V_{LG1} = \frac{K_{LG}}{(\tau_{LG1}\tau_{LG2})}\log_{10}(10^4 P) \tag{13}$$

If $\dot{V}_{LG1} = V_{LG2}$ and $\ddot{V}_{LG1} = \dot{V}_{LG2}$ then Equation (13) can be written as:

$$\frac{dV_{LG1}}{dt} = V_{LG2} \tag{14}$$

$$\frac{dV_{LG2}}{dt}=\frac{K_{LG}}{(\tau_{LG1}\tau_{LG2})}\log_{10}(10^4 P)-\frac{(\tau_{LG1}+\tau_{LG2})}{(\tau_{LG1}\tau_{LG2})}V_{LG2}-\frac{1}{(\tau_{LG1}\tau_{LG2})}V_{GL1} \tag{15}$$

Logarithmic Rate Amplifier

Logarithmic rate amplifier is also a complex electronic unit which is used for the logarithmic rate power measurement. Logarithmic rate amplifier basically uses the voltage signal of logarithmic amplifier.

In frequency domain, similar to logarithmic amplifier, the logarithmic rate amplifier output voltage can be expressed by a linear gain cascaded with a standard second order transfer function given as[10]:

$$\frac{V_{LG1}}{V_{LG2}}=\frac{K_{LGR}\omega_{nLGR}^2}{S^2+2\varsigma_{LGR}\omega_{nLGR}+\omega_{nLGR}^2} \tag{16}$$

where damping coefficient and natural frequency of logarithmic rate amplifier are defined as:

$$\omega_{nLGR}=\frac{1}{\sqrt{\tau_{LGR1}\tau_{LGR2}}}$$

and

$$\varsigma_{LGR}=\frac{1}{2}\frac{(\tau_{LGR1}+\tau_{LGR2})}{\sqrt{\tau_{LGR1}\tau_{LGR2}}}$$

Taking inverse Laplace transform of Equation (16), the logarithmic rate amplifier output voltage can be expressed in terms of following second order differential equations as:

$$\frac{d^2V_{LGR1}}{dt^2}+\frac{(\tau_{LGR1}+\tau_{LGR2})}{(\tau_{LGR1}\tau_{LGR2})}\frac{dV_{LGR1}}{dt}+\frac{1}{(\tau_{LGR1}\tau_{LGR2})}V_{LGR1}=\frac{K_{LGR}}{(\tau_{LGR1}\tau_{LGR2})}V_{LG2} \tag{17}$$

If $\dot{V}_{LGR1}=V_{LGR2}$ and $\ddot{V}_{LGR1}=\dot{V}_{LGR2}$ then Equation (17) can be written as:

$$\frac{dV_{LGR1}}{dt^2}=V_{LGR2} \tag{18}$$

$$\frac{dV_{LGR2}}{dt}=\frac{K_{LGR}}{(\tau_{LGR1}\tau_{LGR2})}V_{LG2}-\frac{(\tau_{LGR1}+\tau_{LGR2})}{(\tau_{LGR1}\tau_{LGR2})}V_{LGR2}-\frac{1}{(\tau_{LGR1}\tau_{LGR2})}V_{GLR1} \tag{19}$$

Linearization of Non-linear PHWR Model

Consider the operation of the reactor at steady state. It is represented by the condition in which power output is constant and the concentration of delayed neutron precursors and moderator level is in equilibrium with steady state power. Consider that the reactor is in steady state at full power operation. Let C_{i0}, H_0, P_0 and Q_{M0} are steady state values of precursor concentration, moderator level, reactor power and moderator flow control signal respectively.

Consider an incremental change in δQ_M to the input of control valve that will result changes in state vectors as δP, δC_i and δH.

Thus, we have:

$$C_i=C_{i0}+\delta C_i,\ H=H_0+\delta H,\ P=P_0+\delta P,\ Q_M=Q_{M0}+\delta Q_M$$

Linearizing Equations (1)-(2), we get the following linearized first order differential equations:

$$\frac{d(\delta P)}{dt}=\frac{\delta\rho}{l}P_0-\frac{\beta}{l}\delta P+\sum\lambda_i\delta C_i \tag{20}$$

$$\frac{d(\delta C_i)}{dt}=\frac{\beta_i}{l}(\delta P)-\sum\lambda_i\delta C_i\quad i=1,2,...,6 \tag{21}$$

Linearizing Equation (3), we get the following linearized first order differential equation:

$$\delta\rho=2K_C\left(\frac{\pi^2}{H_0^3}\right)\delta H$$
$$\frac{dH}{dt}=\frac{K_F}{A_C}\delta L_1 \tag{22}$$

Linearizing Equations (8)-(10), (14)-(15) and (18)-(19), we get the following linearized first order differential equations:

$$\frac{d(\delta L_1)}{dt} = \delta L_2 \tag{23}$$

$$\frac{d(\delta L_2)}{dt} = K_V \delta Q_M - 2\varsigma\omega_n \delta L_2 - \omega_n^2 \delta L_1 \tag{24}$$

$$\frac{d(\delta V_L)}{dt} = \frac{K_L}{\tau_L}\delta P - \frac{\delta V_L}{\tau_L} \tag{25}$$

$$\frac{d(\delta V_{LG1})}{dt} = \delta V_{LG2} \tag{26}$$

$$\frac{d(\delta V_{LG2})}{dt} = \left(\frac{K_{LG}}{(\tau_{LG1}\tau_{LG2})}\frac{\delta P}{P}\times 2.303\right) - \frac{(\tau_{LG1}+\tau_{LG2})}{(\tau_{LG1}\tau_{LG2})}\delta V_{LG2} - \frac{1}{(\tau_{LG1}\tau_{LG2})}\delta V_{GL1} \tag{27}$$

$$\frac{d(\delta V_{LGR1})}{dt} = \delta V_{LGR2} \tag{28}$$

$$\frac{d(\delta V_{LGR2})}{dt} = \frac{K_{LGR}}{(\tau_{LG1}\tau_{LG2})}\delta V_{LG2} - \frac{(\tau_{LGR1}+\tau_{LGR2})}{(\tau_{LGR1}\tau_{LGR2})}\delta V_{LGR2} - \frac{1}{(\tau_{LGR1}\tau_{LGR2})}\delta V_{GLR1} \tag{29}$$

State Space Realization of Linearized PHWR Model

A continuous time state space model of pressurized heavy water reactor is obtained from Equations (20)-(29) in this form:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{30}$$

$$y(t) = Cx(t) \tag{31}$$

where $A \in R^n$, $B \in R^m$ and $C \in R^p$ are the matrices of appropriate dimensions.

Moderator flow control signal can vary from 0 V to 3.3 V for a power variation from 0.1 MW_{th} to 432 MW_{th}. The input vector, state vector and output vector of a PHWR model described in Equations (30) and (31) are as follows:

$$u(t) = Q_M$$

$$x(t) = \begin{bmatrix} \delta P\ \delta C_1\ \delta C_2\ \delta C_3\ \delta C_4\ \delta C_5\ \delta C_6\ \delta V_L \\ \delta V_{LG1}\ \delta V_{LG2}\ \delta V_{LGR1}\ \delta V_{LGR2}\ \delta L_1\ \delta L_2\ \delta Q_M \end{bmatrix}^T$$

$$y(t) = [y_1\ y_2]^T = [V_L\ V_{LGR1}]^T$$

SYNTHESIS OF FAST OUTPUT SAMPLING CONTROLLER

Following are the constraints imposed on the performance of a controller:

(i) Maximum power overshoot < 12%

(ii) Maximum log rate < 4%

Fast Output Sampling Feedback Method

Fast output sampling is a type of multi-rate output feedback, in which the system output is sampled at faster rate as compared to control input. The fast output sampling technique is a very useful technique as states of the model can be estimated from the output of the model.

A pole assignment problem was investigated using fast output sampling method for linear time-invariant systems in[11]. The design procedure for fast output sampling controller was used for practical application in[12]. The implementation of advanced controller on high speed digital computers would be a new possibility in a nuclear power plant.

The pairs (A,B) and (A,C) are assumed to be controllable and observable respectively. A suitable sample time t is selected during which the control signal u is constant. This sampling time is divided into N subintervals of period $D = \tau/N$, $N \geq \nu$, where ν is called the observability index. The output measurements are made at time instants $t = k$D, k = 0,1,2,.... The control signal is applied during the interval $k\tau < t < (k+1)\tau$. The system output is given by:

$$y(t) = [y_1\ y_2 \ldots y_p]^T \tag{32}$$

N number of output samples during the interval between (k-1)th and kth sampling instant are represented as:

$$y(k) = \begin{bmatrix} y(k\tau - \tau \\ y(k\tau - \tau + \Delta) \\ y(k\tau - \tau + 2\Delta) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix} \tag{33}$$

The sampled data control is applied to the system as follows:

$$u(k) = [G_0 G_1 G_2 \ldots G_{N-1}] \begin{bmatrix} y(k\tau - \tau \\ y(k\tau - \tau + \Delta) \\ y(k\tau - \tau + 2\Delta) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix} \tag{34}$$

where the matrix blocks G_i represent output feedback gains.

Equation (34) can be written as:

$$u(k) = Gy(k) \tag{35}$$

The concept of fast output sampling is shown in Figure (2).

Let (A_τ, B_τ, C) represent system discretized at a period τ and (A_Δ, B_Δ, C) represent system sampled at an interval Δ such that observability index ν of system (A_Δ, C) is smallest positive integer satisfying[13]:

$$Rank \begin{bmatrix} C \\ CA_\Delta \\ CA_\Delta^2 \\ \vdots \\ CA_\Delta^{D-1} \end{bmatrix} = Rank \begin{bmatrix} C \\ CA_\Delta \\ CA_\Delta^2 \\ \vdots \\ CA_\Delta^{D} \end{bmatrix}$$

Then the discrete time system is represented as following:

$$x(k+1) = A_\tau x(k) + B_\tau u(k) \tag{36}$$

$$y(k+1) = C_\Delta x(k) + D_\Delta u(k) \tag{37}$$

where

$$C_\Delta = \begin{bmatrix} C \\ CA_\Delta \\ CA_\Delta^2 \\ \vdots \\ CA_\Delta^{N-1} \end{bmatrix}, D_\Delta = \begin{bmatrix} 0 \\ CB_\Delta \\ CA_\Delta B_\Delta \\ \vdots \\ C\sum_{j=0}^{N-2} A_\Delta^j B_\Delta \end{bmatrix}$$

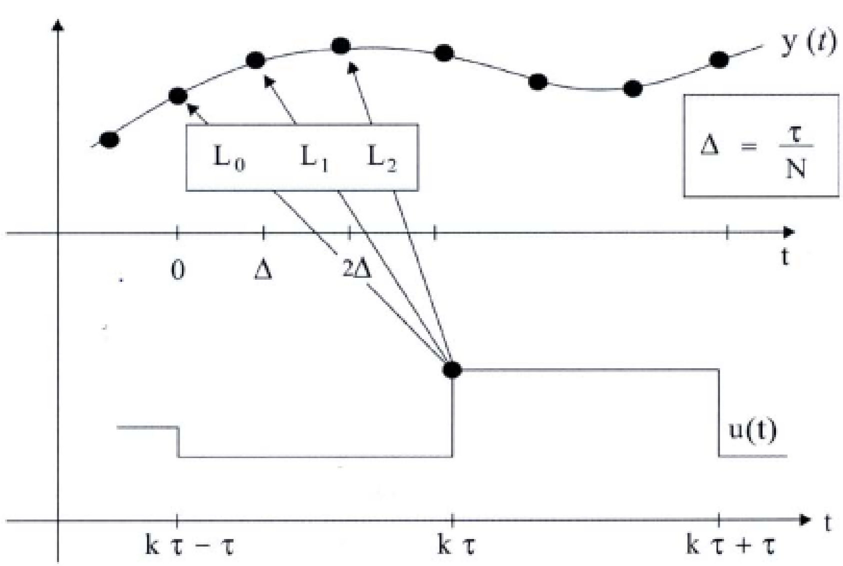

Figure 2: Concept of Fast Output Sampling

As $N \geq v$, C_Δ will be a $pN \times n$ matrix of rank n[14]. Therefore, $C_\Delta^T C_\Delta$ will also be $n \times n$ matrix of rank n and D_Δ will be of $pN \times m$ matrix. Hence, $C_\Delta^T C_\Delta$ would be invertible.

Now consider that the state feedback gain K has been synthesized such that $(A_\tau + B_\tau K)$ has no eigenvalues at the origin. For this state feedback, a fictitious measurement matrix is defined as:

$$C(K,N) = (C_\Delta + D_\Delta K)(A_\tau + B_\tau K)^{-1}$$

Thus, the system output equation can be described as:

$$y(k) = C(K,N)x(k)$$

For G to realize the effect of K, it must satisfy:

$$GC(K,N) = K$$

The schematic representation of fast output sampling control system is shown in Figure (3).

LMI Treatment of the Advanced Controller Design

The fast output sampling controller proposed in Equation (34) will produce the required response but might require unnecessary corrective action. In practical situation, two important problems are required to address. If external disturbance produces large estimation error then the reduction of large estimation error can be computed by the following equation:

$$\phi(G) = GD_\Delta - KB_\tau$$

For stability of control system, all eigenvalues of $\phi(G)$ should be negative and for rapid reduction of large estimation error eigenvalues should be as far as possible from the origin. Another problem is the rise of measurement noise and thus it is desired to keep elements of gain matrix G low.

Instead of exact solution of the controller design problem, we can permit small deviation[7] and use $GC(K,N) \approx K$ which hardly affects the desired closed loop dynamics but may have considerable effect on the two problems described above. If $\gamma_3 = 0$, then G is the exact solution. So, instead of looking for an exact solution to the qualities, the following inequalities are solved:

$$\|G\| < \gamma_1$$
$$\|\phi(G)\| < \gamma_2$$
$$\|GC(K,N) - K\| < \gamma_3$$

Thus, a multi-objective problem has been represented by upper bounds on matrix norms and hence has to be minimized. γ_1 should be minimized for low noise sensitivity, γ_2 should be minimized for rapid reduction of estimation error and γ_3 should be minimized for efficient fast output controller design.

Multi-objective problem can be expressed in the form of Linear Matrix Inequalities as follows[15]:

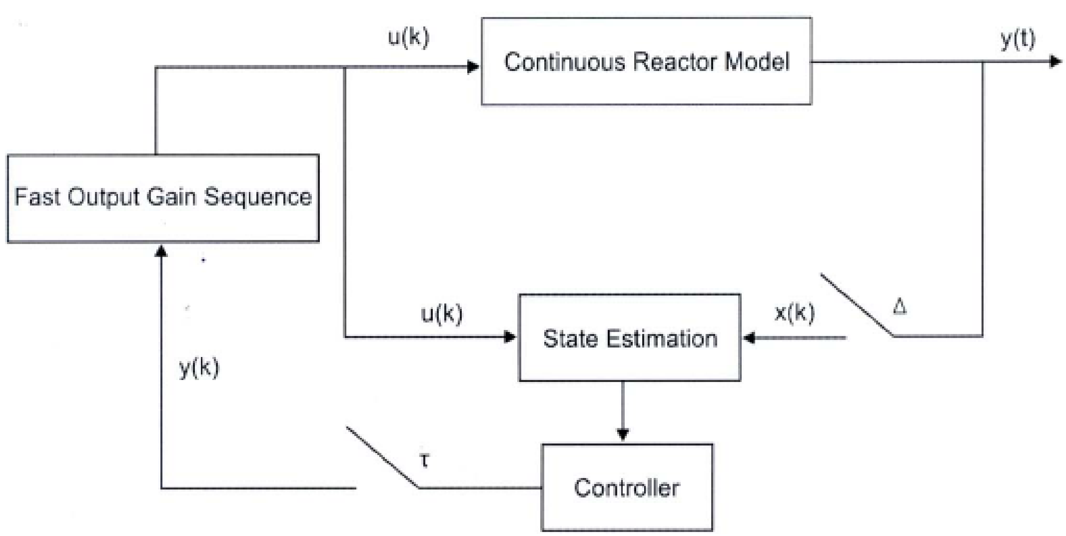

Figure 3: Schematic representation of control system using fast output sampling technique

gain sequence is shown in Table 5. When this state feedback gain sequence is realized using fast output sampling and fast output gain sequence is obtained as explained in section 3.1 then fast output sampling feedback gain sequence so obtained does not address the problems of noise sensitivity and error estimation as explained in section 3.2. Hence, when the problem is posed as LMI formulation as explained in section 3.2, it eradicates the effects of noise and state estimation error keeping the system response almost same as desired. A fast output sampling gain sequence G is designed as explained in Section 3.2 to have the desired closed loop performance of the PHWR power reactor. The designed fast output gain sequence of $m \times pN = 1\times16$ is shown in Table 6. The closed loop eigenvalues of observer based control system are those of $(A_\tau+B_\tau K)$ while closed loop eigenvalues of fast output sampling controller based control system are those of $(GD_\Delta - KB_\tau)$. The closed loop eigenvalues of PHWR using observer based state feedback controller and fast output sampling technique based controller are given in Table 7 and Table 8 respectively. Nevertheless, the comparison shows that there is a very small difference in closed loop eigenvalues with both controllers. All the eigenvalues are negative and lies within the unit circle.

Table 5: State Feedback Gain Sequence (K)

S. No.	K
01	8.206×10^{-2}
02	2.534×10^{-2}
03	3.721×10^{-2}
04	7.890×10^{-2}
05	8.231×10^{-1}
06	4.556×10^{-1}
07	-2.101×10^{-1}
08	6.627
09	9.158×10^{-2}
10	7.871×10^{-2}
11	1.389×10^{-2}
12	2.377×10^{-1}
13	-5.372×10^{-3}
14	-3.485×10^{-4}
15	7.139×10^{-3}

$$\begin{bmatrix} -\gamma_1^2 I & G \\ G^T & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\gamma_2^2 I & \phi(G) \\ \phi^T(G) & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\gamma_3^2 I & GC(K,N)-K \\ (GC(K,N)-K)^T & -I \end{bmatrix} < 0.$$

Since the measurement noise is known so γ_1 can be fixed. Hence, γ_2 and γ_3 has to be minimized. This problem is optimized using the LMI Toolbox for MATLAB[16] and the fast output sampling controller is designed.

APPLICATION OF PROPOSED CONTROLLER TO PHWR

In this section, the proposed advanced controller design methodology is implemented on PHWR model developed in section 2.3 is presented.

The continuous time state space Model obtained from Equations (30) and (31) is discretized for sampling time τ = 1.28 sec and the discretized system (A_τ,B_τ,C) is obtained. Number of subintervals N = 8 are chosen based on the experimental data obtained from an operating nuclear power plant[9]. Thus, a system (A_Δ,B_Δ,C) is obtained for sampling interval Δ = 0.16 sec. From practical experience with PHWR-type nuclear power plant and noise level, the elements of ***G*** with magnitude less than 10^4 can be used[9]. This information about acceptable gain magnitude is used by fixing $\gamma_1 = 10^4$. Upper bounds (γ_2 and γ_3) on the magnitude of the maximum eigenvalue of $\phi(G)$ and the distance between **L** and the exact solution as two conditions are minimized so that observation errors die out fast and well approximate the original state feedback design. Now, the LMI control toolbox for MATLAB is used to minimize a linear combination of γ_1, γ_2 and γ_3. This approach is very useful when the actual measurement noise is known, thus the magnitude of ***G*** is fixed accordingly. Since, in this paper an advanced controller is designed using fast output sampling technique rather than sate feedback controller. So, the main emphasis is on the fast output sampling controller. But for comparison, an observer based state feedback controller is designed and tested under the same conditions. The designed state feedback

Table 6: Fast Output Gain Sequence (G)

S. No.	G
01	-2010
02	3.621×10^{-3}
03	9.030×10^{-5}
04	8.346×10^{-1}
05	9.037×10^{-1}
06	2.213×10^{-3}
07	5.724×10^{-1}
08	4.114
09	3.905×10^{-1}
10	4.527×10^{-1}
11	1.193×10^{-1}
12	1.224×10^{-1}
13	-9.334×10^{-1}
14	-8.776×10^{-1}
15	5.537
16	18.784×10^{-1}

Table 7: Closed Loop Eigenvalues with State Feedback Gains

S. No.	Eigenvalues
01	-3.2531×10^{-1}
02	-5.4734×10^{-1}
03	-4.4710×10^{-1}
04	-3.6678×10^{-1}
05	$-6.5592 \times 10^{-1} + j8.9327 \times 10^{-2}$
06	$-6.5592 \times 10^{-1} - j8.8.9327 \times 10^{-2}$
07	$-9.8849 \times 10^{-1} + j3.4451 \times 10^{-2}$
08	$-8.3159 \times 10^{-1} - j3.4451 \times 10^{-2}$
09	-8.7790×10^{-14}
10	-7.1136×10^{-14}
11	-3.2245×10^{-1}
12	-2.9023×10^{-8}
13	-7.7734×10^{-1}
14	-8.8721×10^{-8}
15	-5.6578×10^{-8}

Table 8: Closed Loop Eigenvalues with Fast Output Sampling Gains

S. No.	Eigenvalues
01	-2.3764×10^{-1}
02	-4.3876×10^{-1}
03	-4.4581×10^{-1}
04	-5.7643×10^{-1}
05	$-5.8354 \times 10^{-1} + j7.6508 \times 10^{-2}$
06	$-5.8354 \times 10^{-1} - j7.6508 \times 10^{-2}$
07	$-8.3159 \times 10^{-1} + j1.2310 \times 10^{-2}$
08	$-8.3159 \times 10^{-1} - j1.2310 \times 10^{-2}$
09	-1.4919×10^{-14}
10	-5.9822×10^{-14}
11	-5.9415×10^{-1}
12	-4.1668×10^{-8}
13	-9.6894×10^{-1}
14	-9.9875×10^{-8}
15	-6.9029×10^{-8}

RESULTS AND DISCUSSION

In this section, dynamics and behavior of the proposed advanced control system is evaluated. In order to simulate the dynamics of the PHWR, the designed fast output gain sequence is substituted in Equation (34) and the so obtained control law is implemented on the discrete time model of PHWR described in Equations (35)-(36).

The PHWR reactor control offers slow response inherently because of large size of calandria. However, this type of slow response of a system is undesired as the corrective steps taken towards decreasing the reactor power will also be limited and the settings of logarithmic rate power and linear power might be exceeded.

When the reactor power is low (less than 13 MW_{th}), the logarithmic rate control loop is dominant in the PHWR reactor. Therefore, during start-up from low power level, the log rate loop controls the reactor. Finally, when the reactor power approaches the demand value, the control is taken over by linear power loop. However, both loops are not controlling independently but the total control signal has contribution from both control loops. The controller is so designed that moderator control valve position is 30.5% open when 100% reactor power is demanded.

In the simulation of proposed advanced control system, it is assumed that the PHWR is operating at 270 MW_{th} and a new power of 310 MW_{th} is dialed. Since the reactor is operating at 270 MW_{th} steady power, so it is in hot condition and no need to add any soluble poison. In power maneuvering from 270 MW_{th} to 310 MW_{th}, the moderator level in the calandria or reactor core would be less than 182 inches, therefore no need of any control rods movement. In cold reactor start-up, the addition of soluble poison slow down the reactivity changes because it is a reactor poison while the movement of control rods in 180 to 186 inches speed up the reactivity changes. This power maneuvering of 40 MW_{th} in reactor hot condition introduces a reactivity change of 0.55 mk. Reactivity is a reactor neutronic parameter and it is a measure departure from the point of criticality. The reactivity of the critical reactor operating at steady power should be 0 mk. When the reactor attains the dialed new power level of 310 MW_{th}, the reactivity of the system becomes 0 mk which proves reactor criticality. The control signal changes from 2.066 Volt to 2.25 Volt. The small variation in control signal shows that proposed control of moderator level is a very fine control and hence it is meant for nuclear reactor power regulation purpose. The moderator level changes from 172.375 inches to 173.625 inches and then attains the previous moderator level of 172.375 inches. It means nuclear fuel has sufficient available burn-up to run the reactor for a longer period of time. The system shows maximum overshoot of 7% and logarithmic rate is well below maximum permissible limit. The setting time for this power transient is 1150 seconds. This settling time is acceptable as one of the most important PHWR transient specifications because a nuclear reactor usually takes large time to settle down the transients otherwise it would move the plant towards uncontrollable conditions and reactor protection system will call upon. The closed loop step response of the variables of interest viz. control signal, moderator valve position, moderator level, reactivity, and reactor power are shown in Figures 4-8. Response of the proposed controller is well within the reactor safe limits.

CONCLUSIONS

Higher order state space model of an operating PHWR power reactor based on point kinetic approach

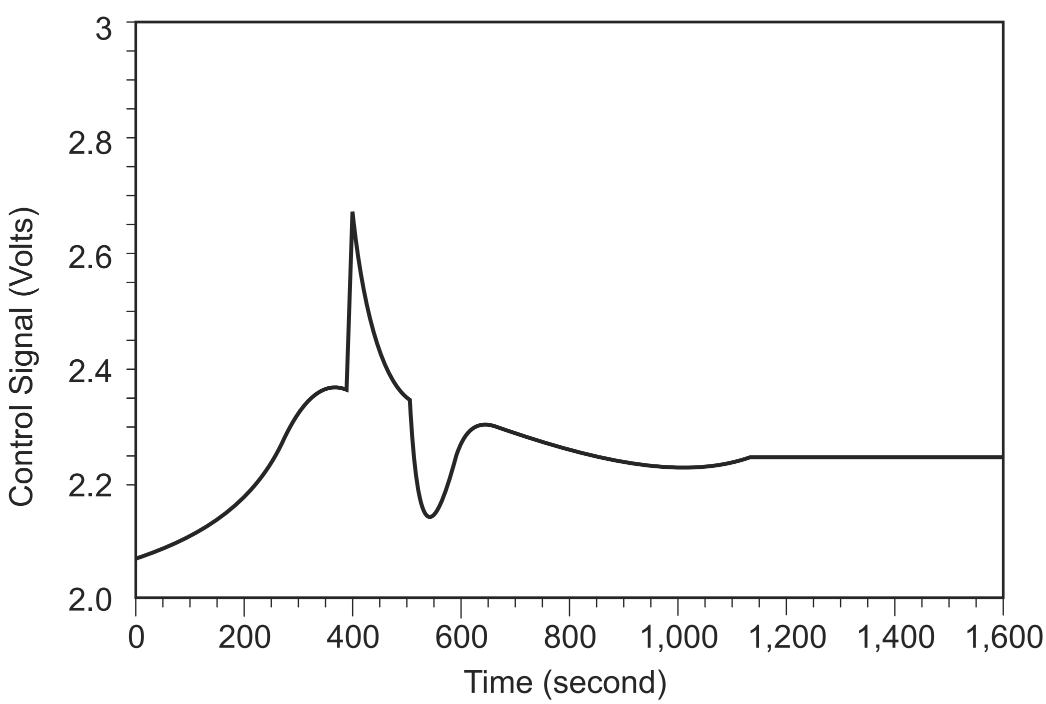

Figure 4: Closed loop dynamics of control valve signal during maneovering from 270 MW_{th} to 310 MW_{th}

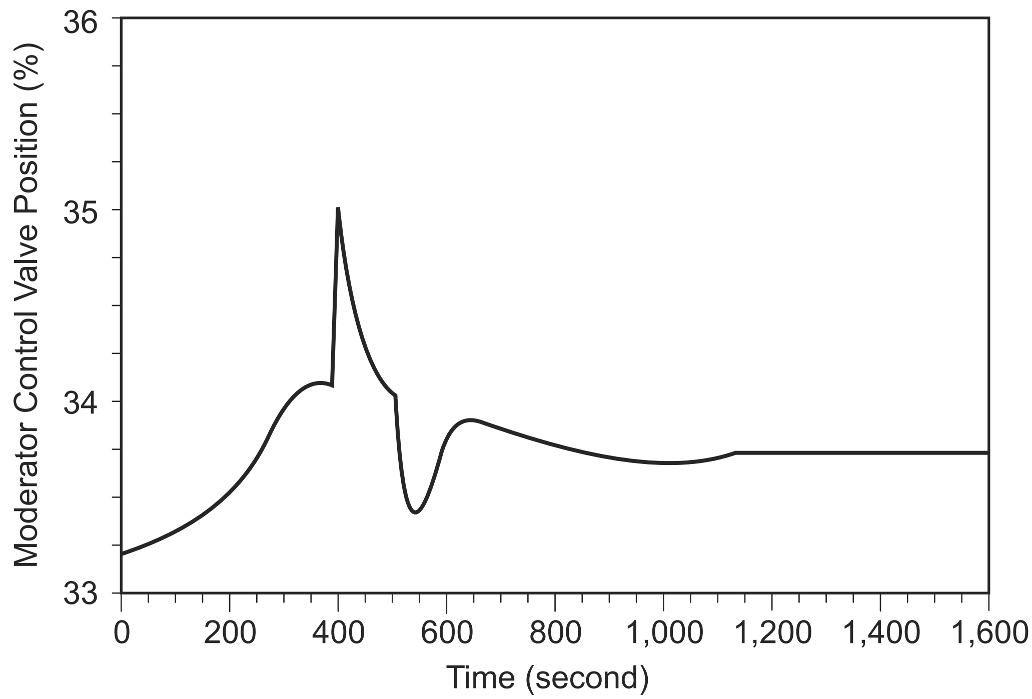

Figure 5: Closed loop dynamics of moderator valve position during maneovering from 270 MW_{th} to 310 MW_{th}

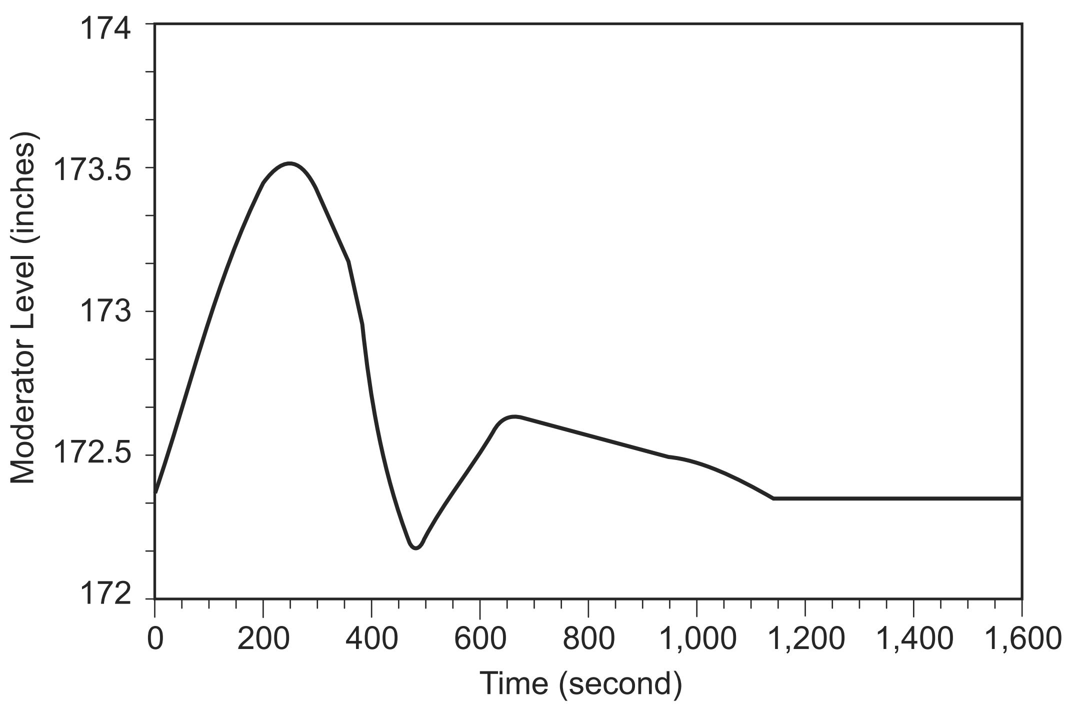

Figure 6: Closed loop dynamics of moderator level duiring maneovering from 270 MW_{th} to 310 MW_{th}

has been developed. A controller has been designed using fast output sampling technique for discrete model of the PHWR power reactor. The method involves finding a stabilizing state feedback for the system to achieve the desired closed loop behavior. Since, this feedback cannot be implemented directly due to some state not being available or measureable or very expensive / complex to measure, the same

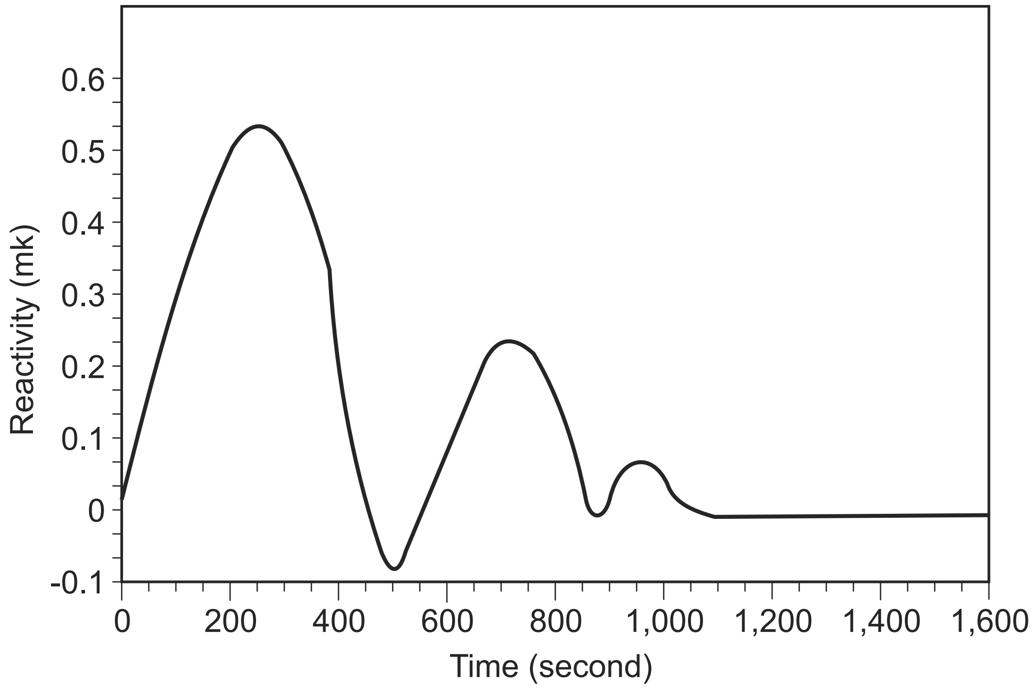

Figure 7: Closed loop dynamics of reactivity maneovering from 270 MW_{th} to 310 MW_{th}

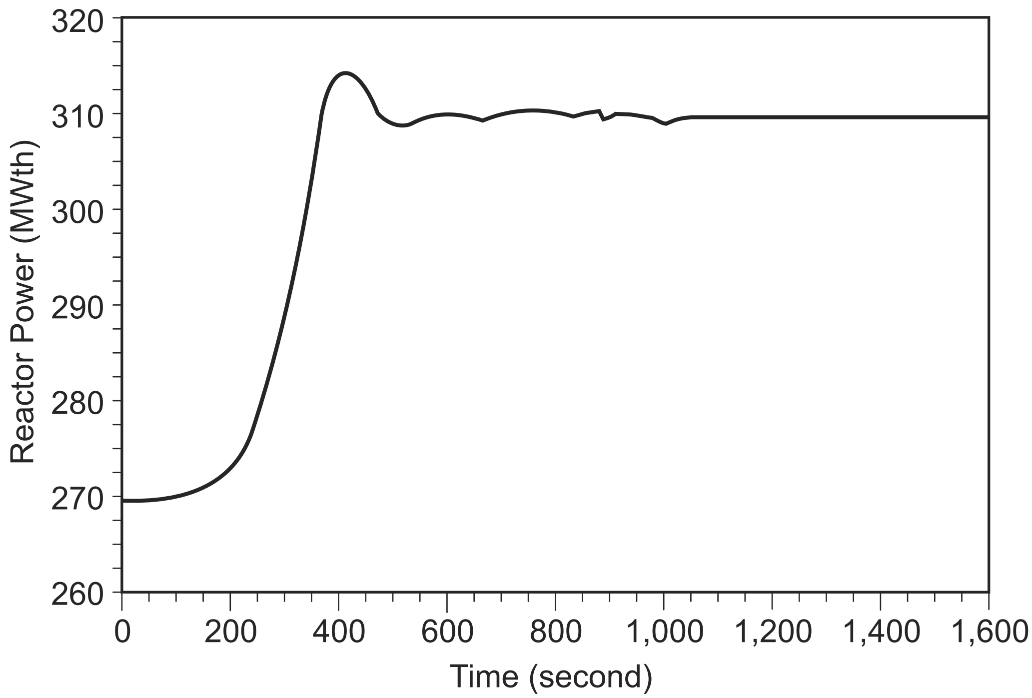

Figure 8: Closed loop dynamics of reactor power during maneovering from 270 MW_{th} to 310 MW_{th}

realized using fast output sampling methodology. Design problem has been reformulated as an LMI problem to overcome the undesired effects like noise sensitivity and estimation error dynamics. The performance of proposed controller has been evaluated under transient condition. Closed loop dynamics of proposed advanced controller is quite satisfactory, fast and efficient.

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