Research Article

Comparing Forecasts of GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs Model: A Study Based On Stock Market Data

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Abstract: Portfolio managers, investors and policymakers are interested in predicting volatility to achieve higher profits or lower risk positions. Due to economic globalization and advancement in information technology, the financial market has become increasingly unstable therefore to understand the pattern of the volatility is difficult. This study aims to compare the performance of Artificial Neural Network (ANNs) and the volatility models Generalized Autoregressive Conditional Heteroscedastic in Mean (GARCH-M) to forecast the stock markets data. The data of two stock markets namely, *Karachi Stock Exchange 100 (*KSE-100) of Pakistan and Standard and Poor's 500 (S and P 500) of USA stock market covering the period 1st January 2013 to 31st, December 2019 are considered for analysis. Various forms of GARCH-M models are applied by inserting conditional standard deviation, conditional variance, or conditional log variance in the conditional mean equation. Moreover, enhancement in the forecasting performance by combining the GARCH-M and ANNs (hybrid model), developed GARCH-M-ANNs model could be seen clearly. As per Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (SBIC), the study shows that GARCH (1, 1)-M estimations by changing conditional mean equations are found to be the most appropriate model. The three measures criterion namely: root mean squares error (RMSE), mean absolute error (MAE) and relative mean absolute error (RMAE) are used for model robustness measurement. The estimation results show that both in-sample and out-sample RMSE, MAE and RMAE are minimum in GARCH (1, 1)-M-ANNs models. Furthermore, empirical analysis reveals that the forecast result of GARCH (1, 1)-M-ANNs models of all three forms give a similar result. The results obtained here are useful for the market practitioners, policymakers and investors.

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Introduction

Volatility plays a key role in derivative pricing and hedging, risk management and optimal portfolio selection. Modeling and forecasting stock market data are always challenging for market practitioners and researchers. Past literatures show that financial return series contains different characteristics such as: Volatility clustering, leverage effect, and long persistence etc.

Furthermore, it has been observed that there are some periods or points where volatility is higher than other periods/points is called leverage effect which occurs due to some economic turbulence such as: Financial crises, Government policies, etc. To model such phenomenon, (Bollerslev, 1986) proposed Generalized Autoregressive Conditional Heteroscedastic (GARCH) model and extension of Autoregressive Conditional Heteroscedastic (ARCH) model by (Engle, 1982). Various extensions in GARCH model are proposed such as: ARCH-M model of (Engle *et al*., 1987; Tsay, 1987), Conditional Heteroscedastic Autoregressive Moving Average (CHARMA) model by (Tsay, 1987; Glosten *et al*., 1993) and Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) by (Nelson, 1991).

In fact, volatility model not only measures conditional volatility, but the standard deviation of each observation also affects the mean of that observation. Engle *et al*. (1987) extended ARCH model of (Engle, 1982) by incorporating standard deviation/ variance in the mean equation, whereas, the unconditional variance is constant. Consideration of standard deviation/ variance explains the fact that changes in the variance equation reflect the risk-return changes in the mean return equation. There are abundant literatures that explain the relationship between stock volatility and stock returns. No risk returns relationship was found among four Chinese stock exchanges using GARCH-M model while GARCH and EGARCH models are used to estimate conditional volatility (Lee *et al*., 2001). Whereas, Salman (2002) found a positive risk-return relationship in Istanbul Stock Exchange. In another study, MARMA, standard GARCH, GARCH-M, and EGARCH models were built for daily returns of Exchange-Traded Funds (ETF) based on Morgan Stanley Capital International (MSCI) indices by (Dedi and Yavas, 2016). Their results show the existence of significant co-movements of returns among the countries namely; Germany, United Kingdom, China, Russia, and Turkey. Moreover, a positive risk-return relationship is found in the UK stock market.

A series of GARCH models were considered to study the exchange rates volatility of 19 Arab countries by (Abdalla, 2012). Their experimental results reveal that family of GARCH model better-explained both exchange rate volatility and leverage effect (Abdalla, 2012). In another study, the volatility of Bucharest Stock Exchange was examined using monthly, weekly and daily returns. As a result, GARCH-M model was found appropriate to explain the structural changes in volatility for Bucharest Stock Exchange. Furthermore, no relationship was found between the risk-returns and future returns (Panait and Slavescu, 2012). Family of GARCH process including Value at Risk (VaR) model were studied for Macedonian stock market. Moreover, VaR with EGARCH-Student's t-distribution was found appropriate to estimate and forecast the volatility of the market (Bucevska, 2013). GARCH-M model was applied for daily returns of emerging Indian financial market Nifty to measure risk-return relationship by (Banumathy and Azhagaiah, 2015). Empirical analysis shows that risk parameter in mean equation of GARCH-M process does not provide the evidence that expected high return is not likelihood to conditional variance.

Recently, due to advancement in data science research artificial intelligence and machine learning models have been broadly applied to forecast the financial and economic data. The artificial neural networks (ANNs) have been proven effective to solve various prediction problems including regression and classification of anomaly detection. Due to non-parametric nature, ANNs has to be effective in modeling nonlinear data, capture linear and nonlinear relationships between input and output variables. This makes ANNs well suited for volatility prediction and has led to increase research in this area. The volatility forecast models such as: ARCH and GARCH were compared with ANNs for Istanbul Stock Exchange 30 **(**ISE30) by (AkarÄ±m, 2013). Returns series of BP/USD, DEM/USD, JPY/USD, and EUR/USD were modeled and forecasted via the family of GARCH models; ARCH, GARCH, Integrated (IGARCH), GARCH (1, 1)-M and EGARCH models (Dhamija and Bhalla, 2010). Results indicated that ANNs model is superior to family of GARCH model when forecasting performance is compared with ANNs model (Dhamija and Bhalla, 2010). Moreover, ANNs model have been compared with available statistical models by (Charef and Ayachi, 2016; Laily *et al*., 2018). Additionally, Fatima and Uddin (2017a) compared forecasting performance of asymmetric GARCH such as EGARCH and Power Generalized Autoregressive Conditional Heteroscedastic (PGARCH) with ANNs models for KSE-100 and Bombay Stock Exchange Sensex (BSESN). ANNs model performed better in out-sample forecast than asymmetric GARCH (Fatima and Uddin, 2017). Moreover, ANNs can provide models for a wide range of natural and artificial phenomena that are difficult to handle using traditional parametric techniques (Enke and Thawornwong, 2005; Liu *et al*., 2011; Wang *et al*., 2012).

Besides these, from the last two decades a combined modeling (hybrid model) approach of statistical and ANNs models have enhanced the prediction of financial data as well. Integrated systems of random walk (RW)-feed-forward ANNs and random walk-Elman ANNs models were developed to forecast exchange rate by (Adhikari and Agrawal, 2014). A combined approach of back-propagation ANNs with support vector regression (SVR) and support vector machine (SVM) for Financial Times Stock Exchange 100 Index (FTS100), S and P500 and Nikkie 225 daily closing indices were developed by (Al-hnaity and Abbod, 2016). Different combinations of empirical models and ANNs have been applied to improve financial data forecast such as: ARIMA, ARCH/GARCH, EGARCH, APGARCH, GJR and NPGARCH models were combined with ANNs (Zhang, 2003; Fatima and Hussain, 2008; Bildirici and Ersin, 2009; Lahmiri, 2017; Chkili and Hamdi, 2021).

It is noted that forecasting stock market data is more important for risk management and portfolio diversification. Various forecasting methods such as ARIMA, family of GARCH models nonlinear models are employed to forecast the volatility of stock returns. However, the finding of previous studies indicates no single method can be applied uniformly to all markets. In this context, this study directed to inspect the forecasting performance of GARCH-M and, ANNs model of stock markets returns.

Thus, the inspiration of this study has the following perspectives: (i) to utilize GARCH-M models for volatility forecasting and also explore risk-return relationship of KSE-100 and S and P 500 indices, (ii) to build ANNs model for forecasting the selected stock market data and (iii) to develop an integrated financial model by providing estimates of ANNs into GARCH-M models to enhance forecasting performance of considered stock markets (iv) to compare the performance of GARCH-M and hybrid GARCH-M-ANNs models.

The rest of the paper is as follows: Description of methodology includes Artificial Neural Networks and GARCH-M models are given in section 2. Whereas, section 3 deals with data analysis and results of GARCH-M, ANNs and combined model of ANNs and GARCH-M. Finally, outcome of the study has been discussed in section 4.

Materials and Methods

Artificial neural networks (ANNs) model

ANNs being part of machine learning techniques learn to mimic human brain. Nowadays, ANNs are used for different purposes such as function approximation, classification, reviewing data and optimization, etc. by (Versace *et al*., 2004). Due to non-parametric and non-linear properties, ANNs model can easily handle data with errors and find a nonlinear association between model parameters. Additionally, ANNs does not require any prior assumptions about the functional relationship among the variables. The ANNs model has ability to learn by the given input (sample data) and adapt to the features that are offered in the data. This data-driven technique is ideal for many empirical studies where there is no theoretical guidance to suggest an effective data generation process (Fahimifard and Kehkha, 2009). Therefore, ANNs are widely used to predict financial market activity and they are found robust with noisy data as well.

ANNs are basically composed of input units, weights, combination function, nonlinear (activation) functions, learning rule and output. The back propagation method (Zhang *et al*., 1998) has become one of the most extensively applied multilayer network learning procedures. Figure 1 represents a three-layer feed-forward networks consisting of an input, a single hidden and an output layers.

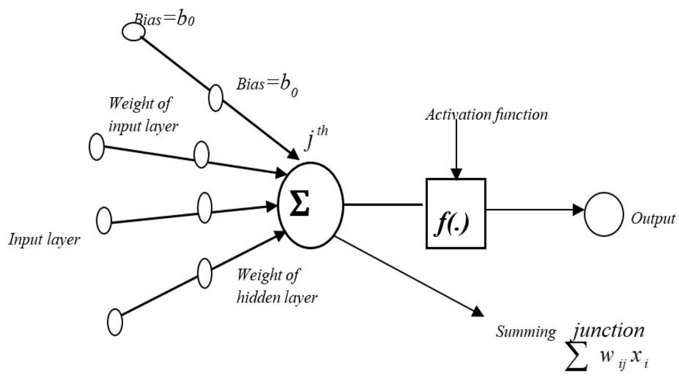

Figure 1: *Artificial Neural Networks, an input layer, a hidden layer and one output layer.*

The relationship of returns is determined via the sum of a linear combination of past lagged values $r_{t-1}, \ldots r_{t-p}$, b_0 (bias), *akj 's* are the weights and *f*(.) is logistic component $(1/1+e^{-y})$ of the lagged values. The relationship between input and output mathematically can be described as:

$$r_t = b_0 + \sum_{k=1}^{p} a_k r_{t-i} + \sum_{j=1}^{q} \bar{a}_j f\left(b_0 + \sum_{k=1}^{p} a_k r_{t-i}\right) + \mathring{a}_t \quad \ldots (1)$$

Equation 1 is called ANNs (*p, d, q*) (Franses and Van Homelen, 1998). Where a_{kj}'s is weights ($k = 1, 2, \ldots, p$, $_j = 1, 2, \ldots, q$) associated with input layers, $\bar{a}_j$ s weights of hidden layers ($_j$=1, 2, …, q), ε_t error of the process, and *p* and *q* represent numbers of nodes in input and hidden layers respectively. The network is trained via small weights with learning algorithms, back-propagation learning is applied which performs well for small problems and provides good result (Stader, 1992).

In this study, integrated models of GARCH-M with ANNs are proposed which are further divided into two steps. In the first step, various ANNs models are developed by changing network structure such as weights, input nodes, hidden layers and hidden nodes, suitable model is selected based on minimum in-sample or test-sample error. While in the second step estimated data from the selected ANNs model are provided into GARCH-M as an input resulting hybrid GARCH-M-ANNs model. Finally, performance of hybrid model is evaluated based on in-sample and out-sample accuracy measures.

GARCH-M model

Modeling and forecasting financial returns series ARCH model was developed by (Engle, 1982) and further extended by (Bollerslev, 1986). An appropriate mean returns equation is required to model the volatility which may be a constant, Autoregressive (AR), moving average (MA) or autoregressive moving average (ARMA) process which is linear and do not have the ability to measure the risk in return equation. To model such phenomena GARCH-M process which inserts conditional standard deviation, conditional variance or conditional log variance in conditional mean equation was proposed (Engle *et al*., 1987).

Let r_t be the returns of a univariate discrete-time stochastic process then conditional mean equation is defined as.

$$r_t = M + e_t \quad \ldots (2)$$

Where, *M* is the expected value of the conditional returns (r_t) and e_t residual of the mean equation of return at time *t* with $E(e_t) = 0$, Var (e_t). Following are the specification of the mean equation of GARCH-M model.

$$r_t = k + \beta(\sigma_t) + e_t \quad \ldots (3) \text{ or}$$

$$r_t = k + \beta(\sigma_t^2) + e_t \quad \ldots (4) \text{ or}$$

$$r_t = k + \beta(\log \sigma_t^2) + e_t \quad \ldots (5)$$

The conditional variance equation of GARCH-(*m*, *n*)- M is:

$$V_t^2 = \theta_0 + \sum_{g=1}^{n} \theta_g e_{t-g}^2 + \sum_{s=1}^{m} \varpi_s V_{t-s}^2 \quad \ldots (6)$$

In Equations 3, 4 and 5, *k* is a constant and *β* is the risk premium parameter. The significant value of *β* captures the influence of volatility of a stock market returns. Where, the sum of the $\theta_g + \bar{\omega}_s < 1$ satisfy the condition of stationary process.

Data analysis

This study utilizes daily closing series of KSE-100 of Pakistan and S and P 500 of USA stock market from 1st January, 2013 to 31st December, 2019 obtained from investing.com. The percentage of daily log-returns of each index is employed by log $(x_t/x_{t-1}) \times 100$. Table 1 shows descriptive statistics of the percentage daily log returns of the KSE-100 and S and P 500. KSE-100 has the highest average return followed by S and P 500, respectively (Table 1). Moreover, KSE-100 is highly volatile with a standard deviation of 0.97 as compared to that of S and P 500 i.e. 0.8. The high value of kurtosis shows the distribution is leptokurtic. Whereas, Jarque–Bera statistics confirm non-normality in all considered series. All descriptive statistics display the stylized facts of financial returns. The Augmented Dickey-Fuller (ADF) test confirms that all the series are stationary at the first difference (Table 2).

ANNs model building process

In ANNs modeling, the percentage of logarithmic returns of daily price data is divided into model building and testing (out-sample forecast). Model building period from January 1st, 2013 to September 6th, 2019 are then split into training and validity set

Table 1: *Descriptive statistics of daily logarithmic percentage returns.*

Variables	Average	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Probability
S and P 500	0.0445	4.798	-4.184	0.7931	-0.5246	6.8456	1216.2010	0.0000
KSE-100	0.0479	4.419	-4.765	0.9676	-0.3194	5.4454	488.9488	0.0000

Note: Above calculations are carried out by the authors.

(Fatima and Hussain, 2008; Fatima and Uddin, 2017b). Whereas, data from September 7th, 2019 to December 31st 2019 is used for out-sample forecasting. Zhang *et al.* (1998) suggested that a single hidden layer network is sufficient to model financial series. In this study, we used feed forward network three-layer: Input, hidden and an output layers with back-propagation learning algorithm. Different network architectures were developed by changing input nodes, weights and training periods. During the training, the RMSE (in and out samples) were calculated to compare for selecting the suitable ANNs model. The ANNs model were developed using the software package Mathematica 10. In-sample root mean squares error (IRMSE) and Out-sample (FRMSE) of fitted ANNs model for various training periods are calculated and presented in Table 3. The ANNs model which has minimum in-sample and out-sample is selected as a suitable model. The estimated data are obtained from the selected model which is further used as input to GARCH-M for developing GARCH-M-ANNs (hybrid) model.

Table 2: *Output of ADF test for the log returns series of the KSE-100 and, S and P500*

Test critical values	KSE-100		S and P500	
	t-Statistic	Prob.	t-Statistic	Prob.
	-37.49	0	-21.847	0
1% level	-3.43		-3.434	
5% level	-2.86		-2.863	
10% level	-2.56		-2.568	

Table 3: *In-Sample and Out-Sample (RMSE) of fitted ANNs model to KSE-100 and S and P 500.*

KSE-100			S and P 500		
Training period	IRMSE	FRMSE	Training period	IRMSE	FRMSE
80	214.26	251.01	81	11.32	13.83
90	337.36	356.64	90	17.67	22.74
101	405.12	463.81	101	17.99	22.15
118	344.2	364.5	112	17.99	22.14
128	194.8**	191.45**	130**	10.79**	14.00**
138	473.59	535.47	144	22.91	29.87**
158	335.9	373.4	151	29.34	35.62
178	346.2	363.6	163	12.15	15.9

*Note: **indicates the minimum value of in-sample and out-sample root mean square error.*

GARCH-M and GARCH-M-ANNs models building process

Next, we develop GARCH-M and GARCH-M-ANNs. The model building process of both the models is same only the input is different. In GARCH-M, daily returns of the considered markets is provided as an input whereas, in the combined model the fitted data of ANNs are passed on to GARCH-M. Various order of GARCH-M are developed such as (1, 1), (1, 2), (2, 1) and (2, 2) but suitable order is selected based on AIC and SBIC.

Table 4 shows the output of ARCH-LM test to check whether ARCH effect is present in the returns series or not. The P-value of the test suggests that we reject the null hypothesis of no ARCH effect is present at 5% level of significance. Furthermore, the result indicates that ARCH effect is present in returns series under consideration, GARCH model is applicable.

Table 4: *Output of ARCH–LM test of the series for the selected stock prices.*

Variables	Models	F statistic	$T^* R^2$	P value
KSE-100	GARCH-M	13.82	53.82	0.00
	GARCH-M-ANNs	21.42	82.07	0.00
S and P 500	GARCH-M	56.87	202.86	0.00
	GARCH-M-ANNs	145.4076	251.3487	0.00

Note: Above calculations are carried out by the authors.

The data from January 1st, 2013 to September 6th, 2019 are used for model building while the data spanned from September 7th, 2019 to December 31st 2019 are used for out-sample forecasting. Therefore, various form of GARCH-M models such as standard deviation ($\sqrt{\sigma^2}$), variance (σ^2) and log variance ($\log\sigma^2$) are employed in conditional mean equation to investigate the question in which of the selected markets an increase in volatility leads to a rise in future returns. GARCH (1, 1)-M is found suitable, outputs are reported in Tables 5, 7 and 9.

Table 5, shows coefficient of the conditional standard deviation($\sqrt{\sigma^2}$) in the mean equation, which is found positive in both types of models, statistically significant at 5% level, demonstrating that an increase in volatility leads to rise in future returns of the both stock markets. However, in the variance equation the coefficients of ARCH is high in S and P 500 i.e. 0.2 in both processes while in KSE-100 it varies from 0.11 to 0.15, indicating that short run shock is high in S and P 500 as compared to KSE-100. Whereas, the coefficients of GARCH in both processes are high in KSE-100 (0.82 and 0.76) and low in S and P500 (0.728 and 0.707) (Table 5), KSE-100 is highly persistent indicating long run shock is high in KSE-100. The sum of coefficients of ARCH and GARCH is less than one indicating shock may be persistent in the future period with slow mean reverting process in both processes.

Furthermore, the validity of both the models for each series is accessed by employing the ARCH-LM test on squares of residuals which exhibits that no ARCH effect is present suggesting that the variance equation is well specified, (Table 6).

Next, the GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs models are estimated by allowing conditional variance as a function in the mean equation of the return series. Table 7 presents the estimation results of both the models; estimated coefficients of risk premium are positive, statistically significant in the mean equation for both stock markets. This designates that the mean of return series significantly depends on past innovation and past conditional variance. Whereas, the results of GARCH (1, 1)-M indicates that when volatility increases the returns are accordingly increased by a factor of 0.12 and 0.13 for KSE-100 and S and P 500, respectively. In contrast, the results of GARCH (1, 1)-M-ANNs demonstrates that when volatility increases the returns are accordingly increased by a factor of 0.174 and 0.134 for KSE-100 and S and P 500, respectively. Whereas, in the variance equation the coefficient of ARCH is high in S and P 500 i.e. 0.2 to 0.24 in both the processes while in KSE-100 it has 0.11 to 0.15 indicating that the short-run shock is high in S and P 500 as compared to KSE-100. However, the coefficient of GARCH is also high in KSE-100 (0.82 and 0.769) and low in S and P500 (0.729 and 0.707) (Table 7), KSE-100 is found to be highly persistent indicating that the long-run shock is high in KSE-100. The sum of coefficients of ARCH and GARCH is less than one indicating that the shock may be persistent in the future period with a slow mean-reverting process.

Furthermore, the validity of the model for each series is assessed by employing the ARCH-LM test on squares of residuals which exhibit that no ARCH effect is present suggesting that the variance equation is well specified (Table 8).

Table 5: *Output of GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs using $\sqrt{\sigma^2}$ in Mean Equation.*

Country	Variable	GARCH (1, 1)- $\sqrt{\sigma^2}$ -M					GARCH (1, 1)- $\sqrt{\sigma^2}$ - M-ANNs				
		Mean equation		Variation equation			Mean equation		Variation equation		
		$\sqrt{\sigma^2}$	C	C	$e^2(-1)$	$V^2(-1)$	$\sqrt{\sigma^2}$	C	C	$e^2(-1)$	$V^2(-1)$
KSE-100	Coefficient	0.247	-0.129	0.060	0.117	0.820	0.337	-0.176	0.062	0.149	0.765
	S. E	0.112	0.098	0.009	0.012	0.016	0.115	0.085	0.008	0.015	0.020
	Z-stat	2.206	-1.317	7.082	9.842	52.104	2.934	-2.05	7.731	9.665	39.163
	Probability	0.027	0.188	0.000	0.000	0.000	0.003	0.040	0.000	0.000	0.000
S and P 500	Coefficient	0.252	-0.080	0.048	0.205	0.728	0.235	0.000	0.000	0.247	0.707
	S. E	0.080	0.052	0.005	0.019	0.021	0.075	0.000	0.000	0.020	0.022
	Z-Statistic	3.154	-1.547	9.244	11.023	34.073	3.152	-0.851	7.309	12.340	32.459
	Probability	0.002	0.122	0.000	0.000	0.000	0.002	0.395	0.000	0.000	0.000

Note: Above calculations are carried out by the authors using Eviews @10 software.

Table 6: *ARCH-LM test of GARCH (1, 1)- $\sqrt{\sigma^2}$ -M and GARCH (1, 1)- $\sqrt{\sigma^2}$ -M-ANNs.*

Variable	Model	F-statistic	Obs*R-squared	Prob. F	Prob. Chi-Square
KSE-100	GARCH (1, 1)- $\sqrt{\sigma^2}$ -M	0.485	1.457	0.6927	0.6922
	GARCH (1, 1)-M- $\sqrt{\sigma^2}$ -ANNs	2.04	6.118	0.1061	0.106
S and P500	GARCH (1, 1)- $\sqrt{\sigma^2}$ -M	0.299	0.6	0.741	0.7406
	GARCH (1, 1)-M- $\sqrt{\sigma^2}$ -ANNs	0.705	2.11	0.5489	0.5484

Note: Above calculations are carried out by the author(s) using Eviews @10 software.

Table 7: *Output of GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs using σ² in mean equation.*

Country	Variable	GARCH (1, 1)- σ²-M					GARCH (1, 1)- σ²-M-ANNs				
		Mean equation		Variation equation			Mean equation		Variation equation		
		σ²	C	C	e²(-1)	V²(-1)	(σ²)	C	C	e²(-1)	V²(-1)
KSE-100	Coefficient	0.120	-0.009	0.061	0.117	0.821	0.174	-0.025	0.061	0.146	0.769
	S. E	0.056	0.048	0.009	0.012	0.016	0.067	0.041	0.008	0.015	0.019
	Z-stat	2.147	-0.19	7.069	9.856	52.047	2.619	-0.61	7.671	9.620	39.605
	Probability	0.032	0.846	0.000	0.000	0.000	0.009	0.544	0.000	0.000	0.000
S and P500	Coefficient	0.135	0.020	0.048	0.203	0.729	0.1348	0.001	0.000	0.244	0.707
	S. E	0.044	0.024	0.005	0.019	0.021	0.5484	0.000	0.000	0.020	0.022
	Z-Statistic	3.053	0.858	9.219	10.986	34.105	2.458	2.687	7.357	12.221	32.421
	Probability	0.002	0.391	0.000	0.000	0.000	0.014	0.007	0.000	0.000	0.000

Note: Above calculations are carried out by the authors using Eviews @10 software.

Table 8: *ARCH-LM test of GARCH (1, 1)- σ²-M and GARCH (1, 1)- σ²-M-ANNs*

Variable	Model	F-statistic	Obs*R-squared	Prob. F	Prob. Chi-Square
KSE-100	GARCH(1, 1)- σ²-M	0.471	1.415	0.7025	0.702
	GARCH(1, 1)- σ²-M -ANNs	2.04	6.104	0.1067	0.1066
S and P500	GARCH(1, 1)- σ²-M	0.328	0.656	0.7206	0.7203
	GARCH(1, 1)- σ²-M -ANNs	0.698	2.1	0.5527	0.5521

Note: Above calculations are carried out by the author(s) using Eviews @10 software.

Table 9: *Output of GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs model using log(σ²) in mean equation.*

Country	Variable	GARCH (1, 1)-log(σ²)-M					GARCH (1, 1)-log(σ²)-M-ANNs				
		Mean equation		Variation equation			Mean equation		Variation equation		
		log(σ²)	C	C	e²(-1)	V²(-1)	log(σ²)	C	C	e²(-1)	V²(-1)
KSE-100	Coefficient	0.121	0.124	0.06	0.118	0.82	0.152	0.17	0.062	0.151	0.762
	S. E	0.052	0.028	0.009	0.012	0.016	0.046	0.036	0.008	0.016	0.02
	Z-stat	2.313	4.414	7.108	9.858	52.29	3.331	4.708	7.801	9.771	39.03
	Probability	0.021	0	0	0	0	9E-04	0	0	0	0
S and P 500	Coefficient	0.098	0.177	0.048	0.206	0.726	7E-04	0.009	0	0.25	0.705
	S. E	0.03	0.034	0.005	0.019	0.021	2E-04	0.002	0	0.02	0.022
	Z-Statistic	3.23	5.165	9.267	11.11	34.14	3.674	4.002	7.36	12.54	32.62
	Probability	0.001	0	0	0	0	2E-04	1E-04	0	0	0

Note: Above calculations are carried out by the authors using Eviews @10 software.

Next, the GARCH (1, 1)-M and GARCH (1, 1)-MANNs models are estimated by allowing conditional log variance as a function in the mean equation of the return series. Table 9, presents the estimation results of the both models, the risk measuring parameter in conditional mean equations have a positive value that indicates the risk-returns relationship is positive, statistically significant in the mean equation for both stock markets at 10% level. This designates that the mean of return series significantly depends on past innovation and past conditional variance. Moreover, in the variance equation, the coefficient of ARCH is high in S and P 500 i.e. 0.2 to 0.25 in both the processes while in KSE-100 it varies from 0.11 to 0.152 indicating that the short run shock is high in S and P 500 as compared to KSE-100. However, the coefficient of GARCH is high in KSE-100 (0.82 and 0.762) and low in S and P500 (0.726 and 0.705), that is, KSE-100 is highly persistent indicating long-run shock is high in KSE-100. The sum of coefficients of ARCH and GARCH is less than one indicating that the shock may be persistent in the future period in both markets with a slow mean-reverting process.

Furthermore, the validity of the model for each series is assessed by employing the ARCH-LM test on squares of residuals which exhibits that no ARCH effect is present suggesting that the variance equation is well specified (Table 10).

Table 10: *ARCH-LM test of GARCH(1, 1)-log(σ²)-M and GARCH(1, 1)- log(σ²)-M-ANNs models.*

Variable	Model	F-statistic	Obs*R-squared	Prob. F	Prob. Chi-Square
KSE-100	GARCH (1, 1)-log(σ²)-M	0.522	1.568	0.667	0.666
	GARCH (1, 1)-log(σ²)-M-ANNs	2.18	6.54	0.11	0.1
S and P 500	GARCH (1, 1)-log(σ²)-M	0.2722	0.545	0.7617	0.7614
	GARCH (1, 1)-log(σ²)-M-ANNs	0.657	1.9744	0.578	0.5777

Note: Above calculations are carried out by the author(s) using Eviews @10 software.

Table 11: *In-sample and out-sample accuracy measures of GARCH (1, 1)-M and GARCH (1, 1)-M-ANNs.*

Model		GARCH (1, 1)-√σ²-M	GARCH (1, 1) - σ²-M	GARCH (1, 1)- log(σ²)-M	GARCH (1, 1)-√σ²- M-ANNs	GARCH (1, 1)-σ²-M-ANNs	GARCH (1, 1)-log(σ²)-M-ANNs
KSE-100	IRMSE	347.719	347.603	347.824	295.35	295.27	295.39
	IMAE	239.83	239.811	239.83	206.70	206.75	206.39
	IMRAE	0.68	0.68	0.684	0.5940	0.5940	0.5900
	FRMSE	483.79	483.72	483.95	402.45	402.73	402.32
	FMAE	391.38	391.45	391.46	321.52	322.13	321.12
	FMRAE	1.00	1.00	1.00	0.83	0.83	0.83
S and P 500	IRMSE	18.33	18.32	18.34	14.12	14.11	14.05
	IMAE	12.27	12.27	12.28	9.53	9.53	9.50
	IMRAE	0.55	0.55	0.55	0.42	0.42	0.42
	FRMSE	13.18	13.16	13.19	10.56	10.54	10.57
	FMAE	10.12	10.10	10.16	8.69	8.65	8.70
	FMRAE	0.32	0.32	0.32	0.27	0.27	0.28

Note: IRMSE (in-sample RMSE), IMAE (in-sample mean absolute error), IRMAE (in-sample mean relative absolute error) and FRMSE (out-sample RMSE), FMAE (out-sample mean absolute error) and FRMAE(out-sample relative mean absolute error).

Table 12: *Comparison of proposed work and other related works discussed in section 3.7 with the state of the art works.*

Data and reference	Models	Results
Lee *et al*., 2001, Chinese stock exchanges	GARCH, EGARCH and GARCH-M	No risk returns relation is found.
(Salman, 2002), Istanbul Stock Exchange	GARCH-M	Positive risk-return relationship
(Dedi and Yavas, 2016), Exchange-Traded Funds based on Morgan Stanley Capital International	MARMA, standard GARCH, GARCH-M and EGARCH	Positive risk-return relationship is found in the UK stock market
(Abdalla, 2012), Nineteen Arab countries	Family of GARCH	volatility and leverage effect were present in exchange rate
(Panait and Slavescu, 2012), Bucharest Stock Exchange	GARCH-M	No risk-returns relationship is found
(Bucevska, 2013), Macedonian Stock market	Family of GARCH with VaR model	EGARCH-Student's t-distribution was appropriate
(Banumathy and Azhagaiah, 2015), Indian stock market	GARCH-M	No risk returns relation is found
(AkarÄ±m, 2013), Istanbul Stock Exchange 30	ARCH and GARCH	volatility forecast
(Dhamija and Bhalla, 2010), BP/USD, DEM/USD, JPY/USD, and EUR/USD	Family of GARCH and ANNs	ANNs model is superior to family of GARCH model
(Charef and Ayachi, 2016), daily exchange rates of Tunisia	GARCH and ANNs	ANNs model provided better forecast family of GARCH model
(Fatima and Uddin, 2017), KSE-100 and BSESN	Neural Network model , GARCH and PGARCH	ANNs model performed better forecast than Family of GARCH
(Adhikari and Agrawal, 2014), Exchange Rate	Random walk, ANNs and Elman ANNs	Hybrid model performed than individual model
(Al-hnaity and Abbod, 2016), FTS100, S and P500 and Nikkie 225 daily closing indices	ANNs, SVR and SVM	Hybrid model outperform than single models
Proposed works		
*Karachi Stock Exchange 100 (*KSE-100) of Pakistan and Standard and Poor's 500 (S and P 500) of USA stock market	Comparison of GARCH-M and hybrid GARCH-M-ANNs	Positive risk-return relationship and better forecast

Overall, results obtained from the various forms of GARCH-M models are approximately same. Most importantly, using various measures such as root mean square (RMSE), mean absolute error (MAE) and mean absolute relative percentage error (MARE) for in-sample and out-sample are calculated and reported in Table 11.

According to Table 11, IRMSE, IMAE and IMRAE of GARCH (1, 1)-M and GARCH-(1, 1)-M-ANNs are minimum indicating developed model is well specified. Furthermore, both the in-sample and out-sample measures are not significantly different using various forms of mean equation (standard deviation, variance and log variance). To put it in a nutshell, GARCH-M (1, 1)-ANNs has minimum in-sample and out-sample errors as compared to GARCH (1, 1)-M.

Table 12, provide a comparison of related works and our proposed model. Past studies show GARCH, EGARCH and GARCH-M models are used to model volatility, risk-return relationship and leverage effect of financial data. Furthermore, forecasting performance of GARCH and EGARCH models compared with ANNs. In this study we proposed hybrid model GARCH-M-ANNs in which modeled data from ANNs was incorporated into GARCH-M. Our developed hybrid GARCH-M-ANNs model not only to measures the risk-return relationship but also to enhance the forecast performance as compared to GARCH-M model.

Conclusions and Recommendations

Volatility is one of the key factors for the market practitioner from the investment point of view. In fact, a stable market attracts investors more than a highly fluctuated market. This study utilizes GARCH-M, ANNs and also combines both models resulting in hybrid model GARCH-M-ANNs which is obtained to forecast USA (S&P500) and Pakistani (KSE-100) stock markets data. In addition, GARCH-M model examines the relationship between mean returns and its variance of the selected stock markets. All the three forms of conditional equations of GARCH-M are studied and GARCH (1, 1)-M is found suitable based on AIC and SBIC values. Moreover, ANNs model is developed and obtained estimated data which is provided as an input to GARCH (1, 1)-M resulting in a hybrid model GARCH (1, 1)-M-ANNs. Empirical analysis shows that the risk parameters in both returns are significant and have positive value indicating that the conditional volatility may increase in the future and investors will be compensated by higher returns for bearing higher levels of risk. Moreover, the estimated parameters of GARCH (1, 1)-M and the hybrid model GARCH (1, 1)-M-ANNs of the conditional mean (standard deviation, variance and log variance) and variance equations are not significantly different. Moreover, the in-sample and out-sample accuracy measures are also same. Furthermore, the GARCH (1, 1)-M-ANNs provide better forecast as compared to GARCH(1, 1)-M model. Due to the nonlinear and nonparametric features of ANNs, it captures the nonlinearity of financial data well and provides better results than the traditional statistical models. Therefore, the hybrid model GARCH (1, 1)-M-ANNs not only provide better forecasts but also providing information about the market behavior. The proposed hybrid model GARCH (1, 1)-M-ANNs could be tested on other markets for further extension and modification of the model such as week days effects and structural change etc.

Novelty Statement

The GARCH-in-Mean model is combined with ANNs to examines the relationship between mean returns and its variance and to forecast conditional volatility.

Author's Contribution

Samreen Fatima: Modelled the data and wrote the manuscript.
Muddasir Uddin: Supervise the study and reviewed the manuscript.

Conflict of interest
The authors have declared no conflict of interest.

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